Benha University Faculty of Engineering- Shoubra Eng. Mathematics & Physics Department Preparatory Year		Final Term Exam Date: June 5, 2016 Course: Mathematics 1 – B Duration: **3** hours
• Answer **All** Questions The Exam consists of one page		• No. of questions: 4 Total Mark: 100

Question 1

(a)Find **y`** : (i) $y = \sinh 2x \,.\, \cosh x^2$ (ii) $y = \tanh x + \ln \operatorname{sech} x$ (iii) $y = \tan^{-1} x^3 + \sinh^{-1} x$ (iv) $y^3 = x^2 + \tanh^{-1}(xy)$ 8

(b)Find the following integrals : 18

(i) $\int (\frac{1}{x^2} + 2^{2x}) dx$ (ii) $\int (\frac{x^2}{1+x^2} + \frac{x}{\sqrt{1+x^2}}) dx$ (iii) $\int \frac{\cosh 3x}{e^x} dx$

(iv) $\int (\frac{3}{x+1} + \frac{1}{\sqrt{x^2-1}}) dx$ (v) $\int \frac{1}{x}(3 + \log x)^8 \, dx$ (vi) $\int (x + \frac{1}{x \ln x}) \, dx$

(vii) $\int x \sin^{-1} x \, dx$ (viii) $\int \cos^2 x \,.\, \sin 3x \, dx$ (ix) $\int \frac{x}{x^2 - 2x + 2} dx$

Question 2

(a)Find the area of the region between the curve : $y = x^3 - 1$, x-axis, x in [0, 2]. 6

(b)Find the arc length of the curve : $y = x^2 + \frac{1}{8} \ln x$, x in [1, 2]. 6

(c)If the region between the curve $y = x^2$, x-axis, x in [1, 2] is rotated about y-axis. Find the volume and the surface area of the generated solid. 6

(d)Find the surface area of the surface generated by rotating, about x-axis, the curve : 6
$x = 2t, \quad y = \ln t - \frac{1}{2} t^2, \quad 1 \le t \le 2.$

Question 3

(a)Find the locus of the middle points of chords of the parabola $y^2 = 4ax$ which passes through (a, 0). 7

(b)Find the value of λ such that the equation $12x^2 - 10xy + 2y^2 + 11x - 5y + \lambda = 0$ represents pair lines. Find the angle between them and the equation of bisectors. 6

(c)Drive the equation of the circle which passes through the origin and cuts off intercepts equal to 4 and 3 from the axes. 6

(d)Prove that the tangents from the point (1, 2) to the circle :
$x^2 + y^2 - 4x + 2y = 0$ are mutually perpendicular. 6

Question 4

(a)Sketch the graph of the equation : $7x^2 - 6\sqrt{3}\, xy + 13y^2 - 16 = 0$ 7

(b)Drive the equation of the parabola whose focus is (2,1) and directrix 3x + 4y = 0 6

(c)Find the equation of the latus rectum, the eccentricity and the coordinate of the foci of the ellipse : $x^2 + 2y^2 + 4x + 4y - 2 = 0$ 6

(d)Find the common tangent to the curves : $y^2 = 8x$ and $xy = -1$ 6

Good Luck *Dr. Mohamed Eid* *Dr. Fathi Abdsallam*

Model Answer

Question 1

(a)(i) $y` = \sinh 2x\,.\sinh x^2\,.\,2x + 2\cosh 2x\,.\cosh x^2$

(ii) $y` = \text{sech}^2 x - \frac{\text{sech } x.\tanh x}{\text{sech } x}$ (iii) $y` = \frac{3x^2}{1+x^6} + \frac{1}{\sqrt{1+x^2}}$

(iv)From $3y^2 y` = 2x + \frac{xy`+y}{1-x^2y^2}$. Then $y` = \frac{2x+\frac{y}{1-x^2y^2}}{3y^2-\frac{x}{1-x^2y^2}}$

-- ***(8 Marks)***

(b) (i) $\int(\frac{1}{x^2} + 2^{2x})dx = -\frac{1}{x} + \frac{1}{\ln 4}4^x + c$

(ii)$\int(\frac{x^2}{1+x^2} + \frac{x}{\sqrt{1+x^2}})dx = \int(\frac{1+x^2-1}{1+x^2} + \frac{x}{\sqrt{1+x^2}})dx = x - \tan^{-1} x + \sqrt{1+x^2} + c$

(iii)$\int \frac{\cosh 3x}{e^x}dx = \frac{1}{2}\int(e^{2x} + e^{-4x})dx = \frac{1}{2}\left(\frac{1}{2}e^{2x} - \frac{1}{4}e^{-4x}\right) + c$

(iv)$\int(\frac{3}{x+1} + \frac{1}{\sqrt{x^2-1}})dx = 3\ln(x+1) + \cosh^{-1} x + c$

(v) $\int \frac{1}{x}(3+\log x)^8\,dx = \frac{\ln 10}{9}(3+\log x)^9 + c$

(vi) $\int(x + \frac{1}{x\ ln\ x})\,dx = \int x\,dx + \int \ln x\,dx = \frac{1}{2}x^2 + \ln\ln x + c$

(vii)Integrate by parts, we get $\int x\sin^{-1} x\,dx = \frac{1}{2}x^2\sin^{-1} x - \frac{1}{2}\int\frac{x^2}{\sqrt{1-x^2}}dx$

By substitution, put x = sin t, we get

$$\int\frac{x^2}{\sqrt{1-x^2}}dx = \int \sin^2 t\,dt = \frac{1}{2}\left(t - \frac{1}{2}\cos 2t\right) = \frac{1}{2}\sin^{-1} x - \frac{1}{4}\cos(2\sin^{-1} x)$$

Then $\int x\sin^{-1} x\,dx = \frac{1}{2}x^2\sin^{-1} x - \frac{1}{2}\left(\frac{1}{2}\sin^{-1} x - \frac{1}{4}\cos(2\sin^{-1} x)\right) + c$

(viii) $\int \cos^2 x\,.\sin 3x\,dx = \frac{1}{2}\int(1+\cos 2x)\sin 3x\,dx$

$$= \frac{1}{2}\int\left(\sin 3x + \frac{1}{2}(\sin x + \sin 5x)\right)dx = -\frac{1}{2}\left(\frac{\cos 3x}{3} + \frac{1}{2}\left(\cos x + \frac{\cos 5x}{5}\right)\right) + c$$

(ix) $\int\frac{x}{x^2-2x+2}dx = \frac{1}{2}\int\frac{2x-2+2}{x^2-2x+2}dx = \frac{1}{2}\int(\frac{2x-2}{x^2-2x+2} + \frac{2}{1+(x-1)^2})\,dx$

$$= \frac{1}{2}\ln(x^2-2x+2) + \tan^{-1}(x-1) + c$$

-- ***(18 Marks)***

Question 2

$$(a)\ A=\int_0^2 (x^3-1)\,dx=\int_0^1 (x^3-1)\,dx+\int_1^2 (x^3-1)\,dx=|-\frac{3}{4}|+\frac{11}{4}=\frac{7}{2}$$

--- ***(6 Marks)***

$$(b)\ L=\int_1^2 \sqrt{1+(2x+\frac{1}{8x})^2}\,dx=3.25$$

--- ***(6 Marks)***

$$(c)\ V_y=2\pi\int_1^2 x^3\,dx=\frac{15}{2}\pi$$

$$S_y=2\pi\int_1^2 x\sqrt{1+(y`)^2}\,dx=2\pi\int_1^2 x\sqrt{1+4x^2}\,dx=\frac{\pi}{6}\left(17\sqrt{17}-5\sqrt{5}\right)$$

$$S=S_y+external\ cylinder+internal\ cylinder+ring$$

$$=\frac{\pi}{6}\left(17\sqrt{17}-5\sqrt{5}\right)+2\pi.2.4+2\pi.1.1+(4\pi-\pi)=\frac{\pi}{6}\left(17\sqrt{17}-5\sqrt{5}\right)+21\pi$$

--- ***(6 Marks)***

$$(d)\ S_x=2\pi\int_1^2 y.\sqrt{(\dot{x})^2+(\dot{y})^2}\,dt=2\pi\int_1^2 (\ln t-\frac{1}{2}t^2).\sqrt{4+[\frac{1}{t}-t]^2}\,dt$$

$$=2\pi\int_1^2 (\ln t-\frac{1}{2}t^2).\sqrt{(\frac{1}{t}+t)^2}\,dt=2\pi\int_1^2 (\ln t-\frac{1}{2}t^2).(\frac{1}{t}+t)\,dt$$

$$=2\pi\int_1^2 (\frac{\ln t}{t}+t\ln t-\frac{1}{2}t-\frac{1}{2}t^3)\,dt$$

$$=2\pi\left(\frac{1}{2}(\ln t)^2+\frac{1}{2}t^2\ln t-\frac{1}{4}t^2-\frac{1}{4}t^2-\frac{1}{8}t^4\right)$$

$$=2\pi\left(\left(\frac{1}{2}(\ln 2)^2+2\ln 2-4\right)+\frac{5}{8}\right)=|-3.5|\pi$$

--- ***(6 Marks)***

Dr. Mohamed Eid